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$$\mathcal{L}\left\{\frac{x^3}{7}e^{4x}\right\} = ? = \frac{6}{(s-4)^4}$$

$$\frac{3!}{s^4} = \frac{6}{s}$$

8- جواب معادله دیفرانسیل $y''+8y'+16y=0$ و $y'(0)=1$ , $y(0)=2$ را بدست آورید

$$\mathcal{L}y'' + \mathcal{L}8y' + 16\mathcal{L}y = 0 \qquad s^2Y - 2s - 1 + 8sY - 16 + 16Y = 0$$

$$Y(s+4)^2 = 2s+17 \Rightarrow Y = \frac{(2s+17)}{(s+4)^2} = \frac{2(s+4)+9}{(s+4)^2}$$

$$y = \mathcal{L}^{-1}Y = e^{-4t}\mathcal{L}^{-1}\frac{2s+9}{s^2} = e^{-4t}(2+9t)$$

-9

$$y'' - 2y' + y = xe^x + 4 \quad , \quad y(0) = y'(0) = 1$$

$$\mathcal{L}y'' - 2\mathcal{L}y' + \mathcal{L}y = \mathcal{L}(xe^x + 4) = \mathcal{L}xe^x + \mathcal{L}4$$

$$s^2\mathcal{L}y - s - 1 - 2sY + 2 + Y = \frac{1}{(s-1)^2} + \frac{4}{s}$$

جزء به جزء

$$Y = \frac{1}{(s-1)^4} + \frac{4}{s(s-1)^2} + \frac{s-1}{(s-1)^2} \Rightarrow \underset{\mathcal{L}^{-1}Y}{y} = e^t\frac{t^3}{3} + e^t + 4\int_0^t te^t\,dt$$

قضیه دوم انتقال: انتقال بر محور $t$:

تابع پله‌ای واحد

$$U_a(t) = U(t-a) = \begin{cases} 1 & t > a \\ 0 & t < a \end{cases}$$

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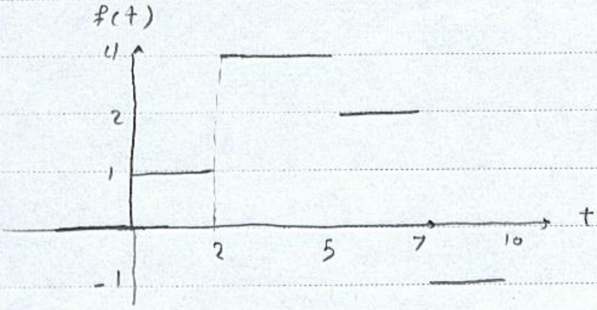
$\int_0^a 0 \cdot e^{-st} dt + \int_a^\infty e^{-st} dt$

$\mathcal{L}(U_a(t)) = \int_a^\infty e^{-st} dt = \frac{1}{s} e^{-as}$

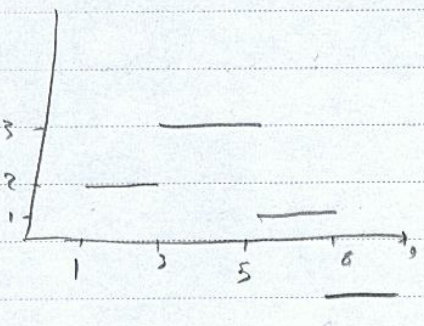
$\mathcal{L}(U_a(t)) = \frac{1}{s} e^{-as}$

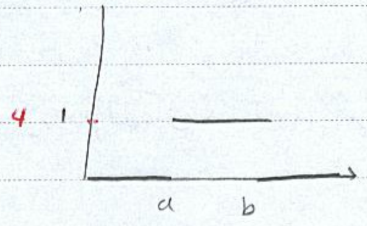



4

$f(t) = U_0(t) + 3U_2(t) - 2U_5(t) - 3U_7(t) + U_{10}(t)$

$2U_1(t) + U_3 - 2U_5 - 3U_8 + 2U_9$




$U_a(t) - U_b(t) = \begin{cases} 1 & a < t < b \\ 0 & \end{cases}$

$f(t) = (U_a(t) - U_b(t)) \frac{1}{4}$

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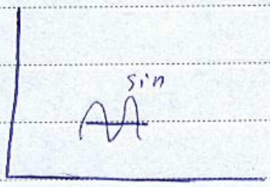


$f(t) = (U_a(t) - U_b(t))\sin t$

$$y'' \_\_\_\_ = f = \begin{cases} t & 0 < t < \pi \\ \cos t & \pi < t < 2\pi \\ (t^2 - 1) & t \geq 2\pi \end{cases}$$

$$\Rightarrow f(t) = (U_0(t) - U_\pi(t))\,t + (U_\pi(t) - U_{2\pi}(t))\cos t + (t^2 - 1)U_{2\pi}(t)$$

$$\mathcal{L}(U_a(t) f(t)) = e^{-as}\,\mathcal{L} f(t+a)$$

$$\mathcal{L} f(t) = e^0 \mathcal{L}(t) - e^{-\pi s}\mathcal{L}(t+\pi) + e^{-\pi s}\mathcal{L}\overbrace{\cos(t+\pi)}^{-\cos t} - e^{-2\pi s}\mathcal{L}\cos(t+2\pi)$$

$$+ e^{-2\pi s}\mathcal{L}((t+2\pi)^2 - 1) = \frac{1}{s^2} - e^{-\pi s}\left(\frac{1}{s^2} + \frac{\pi}{s}\right) - e^{-\pi s}\frac{s}{s^2+1}$$

$$- e^{-2\pi s}\frac{s}{s^2+1} - e^{-2\pi s}\left(\frac{2}{s^3} + 4\pi\frac{1}{s^2} + (4\pi^2 - 1)\frac{1}{s}\right)$$

1- تبدیل لاپلاس

$$f(t) = \begin{cases} t & 0 < t < 2 \\ 2 & t > 2 \end{cases}$$

$$f(t) = \begin{cases} 1 & 0 < t < 2 \\ t & t > 0 \end{cases}$$



$$f(t) = \begin{cases} 2 & 0 < t \leq 1 \\ t & t > 1 \end{cases}$$

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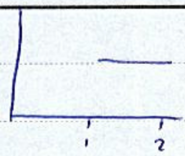
2 - تبدیل لاپلاس



کدام است.

حل 1 قسمت الف :

$$f(t)=\begin{cases} t & 0\le t<2 \\ 2 & t\ge 2 \end{cases} = (U_0-U_2)\,t+2U_2(t)$$

$$\mathcal{L} f(t)= e^{0}\mathcal{L}(t) - e^{-2s}\mathcal{L}(t+2) + 2\frac{1}{s}e^{-2s}$$

$$=\frac{1}{s^2}-e^{-2s}\left(\frac{1}{s^2}+\frac{2}{s}\right)+\frac{2}{s}e^{-2s}$$

3 - معادله لاپلاس $y'+2y=f(t)$ و $y(0)=1$ و $f(t)=\begin{cases} 1 & 1<t<2 \\ 0 & \end{cases}$ را حل کنید؟ کدام است

$\mathcal{L}\,y = Y$

$$sY-1+2Y=\frac{1}{s}\left(e^{-s}-e^{-2s}\right)$$

جواب سوال ← $$Y=\frac{1}{s+2}+\frac{e^{-s}}{s(s+2)}-\frac{e^{-2s}}{s(s+2)}$$

جواب معادله :

مهم $$\left[\mathcal{L}^{-1}\left(e^{-as}F(s)\right)=U_a(t)\,f(t-a)\right] \Leftarrow \mathcal{L}\left(U_a(t)\,f(t)\right)=e^{-as}\,\mathcal{L}\,f(t+a)$$

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$$f(t) = \int_0^t e^{-2t}\,dt = -\frac{1}{2} e^{-2t} \Big|_0^t = \frac{1}{2}(1 - e^{-2t})$$

$$y = e^{-2t} + U_1(t)\,\frac{1}{2}(1 - e^{-2(t-1)}) - U_2(T) \cdot \frac{1}{2}(1 - e^{-2(t-2)})$$

4- تبدیل معکوس $F_s = \frac{2e^{-2s}}{s^2+4}$ هرگاه عامل $e^{-2s}$ آمد باید حتماً از این قضیه استفاده شود (قضیه انتقالی)

$$U_2(t)\sin 2(t-2)$$

سؤال بعد

$$\frac{e^{-\pi s}}{(1+s)^2+1} = U_\pi(t)\sinh 2(t-2)$$

بعدی

$$\frac{2e^{-2s}}{s^2-4} \qquad F_s = \frac{1}{1+(1+s)^2} \Rightarrow f(t) = e^{-t}\sin t$$

$$\rightarrow = U_2(t)\,e^{(t-\pi)}\sin(t-\pi)$$

## مشتق گیری از تبدیل لاپلاس

$$\mathcal{L}f(t) = F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

$$F'(s) = \int_0^\infty e^{-st}(-t f(t))\,dt = \mathcal{L}(-t f(t))$$ مشتق نسبت به s

$$\Rightarrow \mathcal{L}(+t f(t)) = -F'(s)$$

مشتق n ام

$$\Rightarrow \mathcal{L}(t^n f(t)) = (-1)^n F^{(n)}(s)$$

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۱- حاصل انتگرال کدام است

$$\int_0^\infty t e^{-4t} \cos 2t \, dt$$

$$= \mathcal{L}(t\cos 2t)\Big|_{s=4} = -\left(\frac{s}{s^2+4}\right)'\Big|_{s=4} = 0.03 = \frac{3}{100}$$

۲-

$$\int_0^\infty x e^{-sx} \cos \beta x \, dx = \mathcal{L}(x\cos\beta x) = -\left(\frac{s}{s^2+\beta^2}\right)' = -\frac{s^2+\beta^2-2s^2}{(s^2+\beta^2)^2}$$

$$= \frac{s^2-\beta^2}{(s^2+\beta^2)^2}$$

۳-

$$\int_0^\infty t e^{-2t} \sin t \, dt = -\left(\frac{1}{s^2+1}\right)'\Big|_{s=2} = \frac{2s}{(s^2+1)^2}\Big|_{s=2} = \frac{4}{25}$$

۴-

$$\mathcal{L}(t^2 e^{-3t}) = \left(\frac{1}{s+3}\right)''$$

با انتقال $\frac{2}{s^2} \to \frac{2}{(s+3)^3}$

$$\mathcal{L}(t\cos at)$$ ۵-

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۶- لاپلاس رابطه زیر کدام است

$$f(t) = (t-\pi)\, U_{\pi}(t)\, e^{2t} \sin 3t$$

$$= e^{-\pi s} \mathcal{L}\left( t e^{2(t+\pi)} \sin(3t+\pi) \right) = -e^{-\pi s} e^{2\pi} \mathcal{L}\left(e^{2t} t \sin 3t\right) \Rightarrow (1)$$

$$\mathcal{L}(t \sin 3t) = -\left(\frac{3}{s^2+9}\right)' = \frac{6s}{(s^2+9)^2}$$

$$(1) \Rightarrow \; = -e^{-\pi(s-2)}\, 6 \frac{s-2}{((s-2)^2+9)^2}$$

ادامه درس

$$\mathcal{L}(t f(t)) = -F'(s) \qquad \Rightarrow \qquad f(t) = -\frac{1}{t} \mathcal{L}^{-1} F'(s)$$

مورد استفاده در توابع $\ln$ و Arc می باشد.

۱- معکوس $F(s)$ کدام است

$$F(s) = \ln \frac{s-2}{s+2} = \ln(s-2) - \ln(s+2)$$

$$F'(s) = \frac{1}{s-2} - \frac{1}{s+2} \quad \text{معکوس} \Rightarrow \quad e^{2t} - e^{-2t} \quad \cdot \; -\frac{1}{t}$$

$$f(t) = -\frac{e^{2t} - e^{-2t}}{t}$$

2 - 

$$\ln \frac{s}{s-1} = \ln s - \ln(s-1) \qquad F'(s) = \frac{1}{s} - \frac{1}{s-1} \Rightarrow \text{جواب} = \frac{e^{t}-1}{t}$$

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$$F(s) = \ln \frac{(s^2+4\alpha^2)^{1/4}}{s^{1/2}} = \frac{1}{4}\ln(s^2+4\alpha^2) - \frac{1}{2}\ln s$$

$$F'(s) = \frac{1}{2}\left(\frac{s}{s^2+4\alpha^2} - \frac{1}{s}\right)$$

$$\Rightarrow \quad -\frac{1}{2t}\left(\cos 2\alpha - 1\right)$$

۳-

$$F(s) = \cot^{-1} s \qquad F'(s) = \frac{-1}{s^2+1} \quad \Rightarrow \quad \text{جواب} : \frac{\sin t}{t}$$

۴-

$$\mathcal{L}^{-1}\, \frac{1}{s}\, tg^{-1}\frac{1}{s} = \int_0^t \frac{\sin t}{t}\, dt$$

$$\frac{-1/s^2}{1/s^2+1} = \frac{-1}{s^2+1}$$

۵- تبدیل لاپلاس جواب مسأله زیر کدام است.

$$xy'' + (1-x)y' + y = 0 \qquad y(0)=1 \;,\; y'(0) = -1$$

معادله با ضرایب متغیر دو قسمتی آخر

$$\mathcal{L}\, xy'' + \mathcal{L}(1-x)y' + \mathcal{L}\, y = 0$$

Subject:
Year. Month. Date. ( ) $ny' = t^2 \ell t$

$$-(s^2Y - s + 1)' + sY - 1 + (sY - 1)' + Y = 0$$

$$-(2sY + s^2Y' - 1) + sY + \cancel{sY} - 1 + Y + sY' + Y = 0$$

$$Y'(s - s^2) + Y(2 - s) = 0$$

$$\frac{Y'}{Y} = \frac{\cancel{+s+2}}{\cancel{s(1+s)}} - \frac{s-2}{s^2-s} = \frac{-2}{s} + \frac{1}{s-1} \Rightarrow \ln Y = -2\ln s + \ln(s-1)$$

$$= \ln \frac{s-1}{s^2}$$

$$\ln Y = -\frac{1}{2}\ln(s^2 - s) - \frac{3}{2}\ln s + \frac{3}{2}\ln(s-1)$$

$$Y = \frac{s-1}{s^2} \qquad y = 1 - t$$

: 6

$$t y'' + 2y' + ty = 0 \qquad \cdot \tfrac{1}{\pi} = \quad Y = c - y(0)\, tg^{-1} s$$

$$L\, ty'' + 2Ly' + Lty = 0$$

$$+(s^2Y - sy(0) - y'(0))' + 2sY + 2y(0) + Y' = 0$$

$$2sY + s^2Y' - y(0) - 2sY + 2y(0) + Y' = 0$$

$$Y'(s^2+1) = -y(0) \qquad Y' = \frac{-y(0)}{(s^2+1)} \Rightarrow Y = -y(0)\, tg^{-1}(s) + C$$

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انتگرال گیری از تبدیل لاپلاس:

اگر $\lim_{t \to 0^+} \frac{f(t)}{t}$ موجود باشد آنگاه

$$\mathcal{L}\left(\frac{f(t)}{t}\right) = \int_s^\infty F(u)\,du$$

$$\Rightarrow f(t) = t\,\mathcal{L}^{-1}\int_s^\infty F(u)\,du$$

$F(s) = \frac{s}{(s^2+1)^2}$ $\quad \int F(s) = -\frac{1}{2}\,\frac{1}{s^2+1}$ $\quad -\frac{1}{2}\sin t$

$f(t) = -\frac{t}{5}\sin t$

$$\int_0^\infty e^{-x}\,\frac{\sin \alpha x}{x}\,dx = tg^{-1}\alpha$$

$\|$

$$\mathcal{L}(\sin\alpha t)/t = \int_s^\infty \frac{\alpha}{s^2+\alpha^2}\,ds = tg^{-1}\frac{s}{\alpha}\Big|_s^\infty = \frac{\pi}{2} - tg^{-1}\frac{s}{\alpha} = tg^{-1}\frac{\alpha}{s}$$

حفظ شود $\int_0^\infty \frac{\sin u}{u}\,du = \frac{\pi}{2}$ $\quad s=0$ $\quad \mathcal{L}\,\frac{\sin t}{t}$

$\mathcal{L}\,\frac{\sin 2t}{t}$

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در سوال یک بکار رفته $\frac{\pi}{2} - tg^{-1}\frac{1}{\alpha} = tg^{-1}\alpha$

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$\mathcal{L}\ \dfrac{\sin t}{t}$

$\ln \dfrac{e^{t}}{t}$ حد ندارد و جواب نمی‌دهد

$\mathcal{L}\ \dfrac{e^{t}-1}{t} = \int_{s}^{\infty}\left(\dfrac{1}{s-1}-\dfrac{1}{s}\right)ds = \ln(s-1)-\ln s\Big|_{s}^{\infty} = \ln\dfrac{s-1}{s}\Big|_{s}^{\infty}$

$= \ln \dfrac{s}{s-1}$

قضیه پیچش : (کانولوشن)

کانولوشن

$(f * h)(t) = \int_{0}^{t} f(\lambda)\, h(t-\lambda)\, d\lambda$

خاصیت جابجایی دارد

$\dfrac{1}{s(s-1)}$ یا $\dfrac{1}{s^{3}(s^{2}-1)}$ یا $\dfrac{1}{(s^{2}+1)(s^{2}+4)}$

$\mathcal{L}^{-1}\ F(s)\,H(s) = \int_{0}^{t} f(\lambda)\, h(t-\lambda)\, d\lambda$

$F(s)\,H(s) = \mathcal{L}\int_{0}^{t} f(\lambda)\, h(t-\lambda)\, d\lambda$

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$$\overset{1 \quad e^t}{\mathcal{L}^{-1} \frac{1}{s(s-1)}} = \int_0^t e^{\lambda}\, d\lambda$$

$$\overset{\frac{t^2}{2} \quad \sinh t}{\mathcal{L}^{-1} \frac{1}{s^2(s^2-1)}} = \frac{1}{2}\int_0^t (t-\lambda)^2 \sinh\lambda \, d\lambda$$

تبدیل به جمع شوند

$$\mathcal{L}^{-1} \frac{1}{(s^2+1)(s^2+4)} = \frac{1}{2}\int_0^t \sin 2\lambda \, \sin(t-\lambda)\, d\lambda$$

$$\mathcal{L}^{-1} \frac{1}{(s^2+1)^2} = \int_0^t \sin\lambda \, \sin(t-\lambda)\, d\lambda = k(t)$$

$$\mathcal{L}^{-1} \frac{s}{(s^2+1)^3} = \int_0^t k(\lambda) \cos(t-\lambda)\, d\lambda$$

$$\mathcal{L}^{-1} \frac{1}{(s-2)(s+3)^2} = \int_0^t \lambda e^{-3\lambda}\, e^{2(t-\lambda)}\, d\lambda$$

$e^{2t} \cdot te^{-3t}$

۱. جواب $y'' + y = f(x),\ y(0) = y'(0) = 0$ کدام است؟

$$s^2 Y + Y = F(s)$$

$$Y = \frac{1}{s^2+1} F(s) \Rightarrow y = \int_0^x \sin(x-t) f(t)\, dt$$

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۲- جواب معادلهٔ $\varphi(t) = 2 + \int_0^t e^{t-u}\varphi(u)\,du$ کدام است؟

جواب $= 1 + e^{2t}$

$\mathcal{L} \quad = \quad \mathcal{L}$

$$\Phi(s) = \frac{2}{s} + \Phi(s)\frac{1}{s-1}$$

$$\Phi(s)\left(1 - \frac{1}{s-1}\right) = \frac{2}{s} \Rightarrow \Phi(s) = 2\frac{s-1}{s(s-2)}$$

$$\frac{s-2}{s-1}$$

$$\varphi(t) = 1 + e^{2t}$$

۳- جواب معادلهٔ $y(t) = 1 + \int_0^t y(x)\sin(t-x)\,dx$ کدام است؟

جواب $= 1 + \frac{t^2}{2}$ $\quad \mathcal{L} \quad = \mathcal{L}$

$$Y = \frac{1}{s} + \frac{Y}{s^2+1} \Rightarrow Y\left(1 - \frac{1}{s^2+1}\right) = \frac{1}{s}$$

$$Y = \frac{s^2+1}{s^3} \Rightarrow 1 + \frac{t^2}{2}$$

۴- 

$$f(t) = te^{-t} + \int_0^t \alpha f(t-\alpha)e^{-\alpha}\,d\alpha \Rightarrow \text{جواب} = \frac{1}{2}(1 - e^{-2t})$$

تیلور

$$F = \frac{1}{(s+1)^2} + F \cdot \frac{1}{(s+1)^2} \qquad F((s+1)^2 - 1) = 1$$

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صدف حاج آقاسی

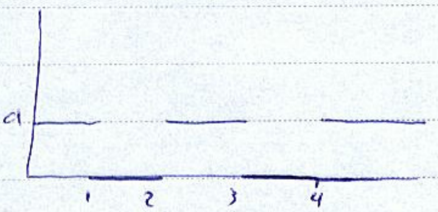
$$F = \frac{1}{(s+1)^2 - 1} \Rightarrow f = \frac{e^{-t}}{2}\left(e^{t} - e^{-t}\right)$$

تابع متناوب:

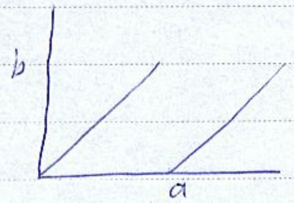
$f(t+p) = f(p)$ ، $p$ کوچکترین عددی باشد که در معادله صدق می کند (دوره تناوب نامند)

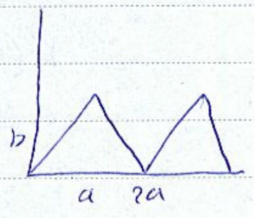
$$\mathcal{L} f(t) = \frac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t)\, dt$$

1-



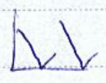

$$\frac{a}{1-e^{-2s}} \int_0^1 e^{-st}\, dt = \frac{a}{s(1-e^{-s})}$$



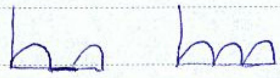




دوره تناوب
عوض می شود

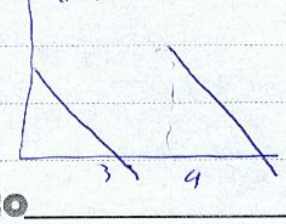
نیم موج

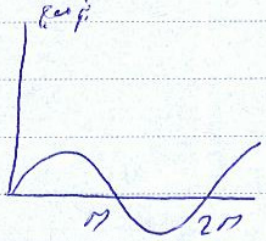


تمام موج



نیم موج



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