* تمرینات فصل دوم (معادلات با مشتقات جزئی)

تمرین ۲. ۵.۲ صفحه‌ی ۹۱
مسئله ارتعاش را در حالتی حل کنید که انحراف در روی مرز ناحیه صفر و سرعت اولیه و انحراف اولیه آن به صورت داده شده باشند.

a. $u(x,0) = x(1-x) \qquad 0 \leq x \leq 1$

$u_t(x,0) = 0$

در این سؤال $\quad l=1 \;,\; \lambda_n = cn\pi \quad \Leftarrow$

$$a_n = \frac{2}{1}\int_0^1 x(1-x)\sin n\pi x \, dx$$

$$a_n = 2\left[(x-x^2)\left(\frac{-\cos n\pi x}{n\pi}\right) - (1-2x)\left(\frac{-\sin n\pi x}{(n\pi)^2}\right) + (-2)\frac{\cos n\pi x}{(n\pi)^3}\right]_0^1$$

$$a_n = \frac{4}{(n\pi)^3}\left[1+(-1)^{n+1}\right] \qquad u_t(x,0)=0 \rightarrow b_n = 0$$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{4}{(n\pi)^3}\left[1+(-1)^{n+1}\right]\cos cn\pi t \,\sin n\pi x$$

b. $u(x,0) = 3\sin x \quad , \quad u_t(x,0) = 0 \qquad 0 \leq x \leq \pi$

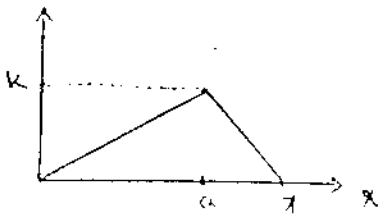
با توجه به $u(x,0)$ نتیجه می‌گیریم بسط سینوسی فوریه این تابع همراه با a_n خواهد بود و تنها شامل جمله‌ی اول بسط با ضریب $a_1=3$ می‌باشد پس سایر جملات ضریب صفر داشته و داریم:

$$\lambda_1 = \frac{c\pi}{\pi} = c$$

$$\Rightarrow u(x,t) = a_1 \sin x \cos ct = 3\cos ct \sin x$$

۲ـ مسئله ارتعاش نخ را در حالتی حل کنید که انحراف اولیه ارتعاش برابر یکی از توابع زیر و سرعت اولیه آن صفر باشد. وضعیت ارتعاش را در چند لحظه که خود انتخاب خواهید نمود به کمک جواب حاصل از روش دالامبر رسم کنید. $(c=1)$

(الف)



$$u(x,0) = \begin{cases} \frac{k}{a} x & 0 \le x \le a \\ \frac{k}{a-\pi}(x-\pi) & a < x \le \pi \end{cases}$$

$u_t(x,0)=0 \qquad u(0,t)=0 \qquad u(\pi,t)=0$

$c=1 \qquad L=\pi \qquad \lambda_n = \frac{cn\pi}{l} = n \qquad b_n = 0$

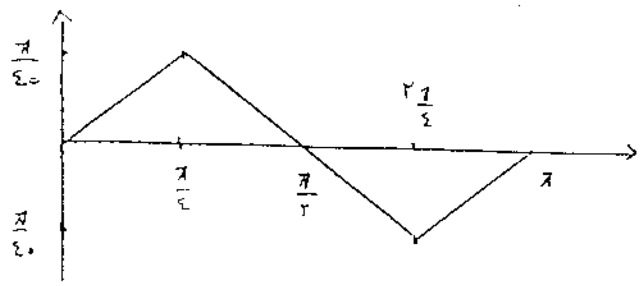
$$a_n = \frac{۲}{\pi}\int_0^a \frac{k}{a} x \sin nx \, dx + \frac{۲}{\pi}\int_a^\pi \frac{k}{a-\pi}(x-\pi) \sin nx \, dx$$

$$= \frac{۲}{\pi}\left[\frac{k}{a} x\left(\frac{-\cos nx}{n}\right) - \frac{k}{a}\left(\frac{-\sin nx}{n^۲}\right)\right]_0^a + \frac{۲}{\pi}\left[\frac{k(x-\pi)}{a-\pi}\left(\frac{-\cos nx}{n}\right) - \frac{k(-\sin nx)}{(a-\pi)n^۲}\right]_a^\pi$$

$$= \frac{۲k}{a(a-\pi)}\left[-\frac{\pi \sin na}{n^۲}\right] = \frac{۲k}{a(a-\pi)} \frac{\sin na}{n^۲} \Rightarrow$$

$$u(x,t) = \frac{۲k}{a(a-\pi)} \sum_{n=1}^{\infty} \frac{\sin na}{n^۲} \cos nt \sin nx$$

ب)



$$u(0,t) = u(\pi,t) = 0$$

$$u_t(x,0) = 0$$

$$u(x,0) = f(x)$$

$$f(x) = \begin{cases} \frac{1}{10}x & 0 < x < \frac{\pi}{4} \\ -\frac{1}{10}\left(x - \frac{\pi}{2}\right) & \frac{\pi}{4} < x < \frac{3\pi}{4} \\ \frac{1}{10}(x-\pi) & \frac{3\pi}{4} < x < \pi \end{cases}$$

$$a_n = \frac{2}{\pi}\int_0^{\frac{\pi}{4}} \frac{1}{10}x \sin nx \, dx + \frac{2}{\pi}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} -\frac{1}{10}\left(x - \frac{\pi}{2}\right)\sin nx \, dx + \frac{2}{\pi}\int_{\frac{3\pi}{4}}^{\pi} \frac{1}{10}(x-\pi)\sin nx \, dx$$

$$= \frac{2}{\pi}\left[-\frac{1}{10}x\frac{\cos nx}{n} + \frac{1}{10}\frac{\sin nx}{n^2}\right]_0^{\frac{\pi}{4}} + \frac{2}{\pi}\left[\frac{1}{10}\left(x - \frac{\pi}{2}\right)\frac{\cos nx}{n} - \frac{1}{10}\frac{\sin nx}{n^2}\right]_{\pi/4}^{3\pi/4}$$

$$+\left[-\frac{1}{10}(x-\pi)\frac{\cos nx}{n} + \frac{1}{10}\frac{\sin nx}{n^2}\right]_{3\pi/4}^{\pi} = \frac{4}{10\pi}\left[\frac{\sin\frac{n\pi}{4}}{n^2} - \frac{\sin\frac{3n\pi}{4}}{n^2}\right]$$

$a_1 = 0 \qquad a_2 = \frac{4}{5\pi} \times \frac{1}{4} \qquad a_3 = 0 \qquad a_4 = \frac{4}{5\pi} \times 0 = 0 \qquad a_5 = 0 \qquad a_6 = \frac{4}{5\pi} \times \left(-\frac{1}{36}\right)$

$a_7 = 0 \qquad a_8 = 0 \qquad a_9 = 0 \qquad a_{10} = \frac{4}{5\pi} \times \frac{1}{100}$

$$\Rightarrow u = \frac{4}{5\pi}\left(\frac{1}{4}\cos 2t \sin 2x - \frac{1}{36}\cos 6t \sin 6x + \frac{1}{100}\cos 10t \sin 10x \mp \cdots\right)$$

۳- جواب هر یک از معادلات زیر را به روش تفکیک متغیرها به دست آورید

a. $u_x + u_y = 0$

$u = g(x)\,f(y)$

$u_x = f(y)\,\dot{g}(x) \qquad u_y = g(x)\,f'(y)$

$u_x + u_y = \dot{g}(x) f(y) + g(x) f'(y) = 0 \implies \dot{g}(x) f(y) = -g(x) f'(y) \implies \dfrac{g(x)}{\dot{g}(x)} = \dfrac{f(y)}{f'(y)} = k$

باید برابر با عدد ثابت باشند

$f(y) + c_0 f'(y) = 0 \implies f(y) = A e^{-\frac{1}{c_0} y}$

$g(x) - c_0 \dot{g}(x) \implies g(x) = A' e^{\frac{1}{c_0} x}$

$\implies U = g(x) f(y) = AA' e^{\frac{1}{c_0}(x-y)} \xrightarrow[\frac{1}{c_0} = c]{AA' = k} k e^{c(x-y)}$

b. $y u_x = x u_y \qquad u = f(x)\,g(y) \qquad u_x = f'(x)\,g(y),\ u_y = \dot{g}(y)\,f(x)$

$y g(y) f'(x) = x f(x) \dot{g}(y) \qquad \dfrac{y\,g(y)}{\dot{g}(y)} = \dfrac{x f(x)}{f'(x)} = k$

$\implies y(g(y)) - k \dot{g}(y) = 0 \quad (1) \qquad x f(x) - k f'(x) = 0 \quad (2)$

$(1) \implies \dfrac{\dot{g}(y)}{g(y)} = \dfrac{y}{k} \implies \ln g(y) = \dfrac{1}{2k} y^2 \implies g(y) = e^{\frac{y^2}{2k}}$

$(2) \implies x f(x) = k f'(x) \implies \dfrac{f'(x)}{f(x)} = \dfrac{x}{k} \implies \ln f(x) = \dfrac{x^2}{2k} \implies f(x) = e^{\frac{x^2}{2k}}$

$u = e^{\frac{1}{2k}(x^2+y^2)} \quad \sim \quad u = e^{k(x^2+y^2)}$

c. $xu_x = yu_y$ $\qquad u_x = f'(x)\,g(y) \qquad u_y = \dot g(y)\,f(x)$

$$\Rightarrow x\frac{f'(x)}{f(x)} = \frac{y\,\dot g(y)}{g(y)} = k \Rightarrow \quad x f'(x) - k f(x) = 0 \quad ①$$

$$y\,\dot g(y) - k\,g(y) = 0 \quad ②$$

$$① \Rightarrow x f'(x) = k f(x) \Rightarrow \frac{f'(x)}{f(x)} = \frac{k}{x} \xRightarrow{\text{انتگرال‌گیری}} \ln f(x) = k\ln x + \ln c_0$$

$$\Rightarrow f(x) = c_0 x^k \quad ①'$$

$$② \Rightarrow y\,\dot g(y) = k\,g(y) \Rightarrow \frac{\dot g(y)}{g(y)} = \frac{k}{y} \Rightarrow \ln g(y) = k\ln y + \ln c_1$$

$$g(y) = c_1 y^k \quad ②'$$

$$①', ②' \Rightarrow u = c\,y^k x^k$$

d. $u_{xy} = u$ $\qquad u_x = f'(x)\,g(y) \qquad u_{xy} = \dot g(y)\,f'(x)$

$$\Rightarrow f'(x)\,\dot g(y) = f(x)\,g(y) \Rightarrow \frac{f'(x)}{f(x)} = \frac{g(y)}{\dot g(y)} = k$$

$$f'(x) = k f(x) \Rightarrow f(x) = e^{kx}$$

$$\Rightarrow u = e^{k(x+y)}$$

$$g(y) = k\,\dot g(y) \Rightarrow g(y) = e^{\frac{y}{k}}$$

e. $u_{xx} + u_x - 2u = 0$

$$D = \frac{du}{dx} \Rightarrow (D^2 + D - 2)\,u = 0 \Rightarrow D = 1 \ , D = -2$$

$$u = A e^{x} + B e^{-2x} \quad ①$$

در ① A و B توابعی از y هستند درنتیجه خواهیم داشت :

$$u = f(y)\,e^{x} + g(y)\,e^{-2x}$$

۴- هر یک از معادلات را با تغییر متغیرهای داده شده حل کنید.

a) $u_{xx} + u_{xy} = 2u_{yy}$ $\qquad v = x+y \qquad z = 2x-y$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial x} = \frac{\partial u}{\partial v} + 2\frac{\partial u}{\partial z}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial v^2}\frac{\partial v}{\partial x} + \frac{\partial^2 u}{\partial v \partial z}\frac{\partial z}{\partial x} + 2\left(\frac{\partial^2 u}{\partial z \partial v}\frac{\partial v}{\partial x} + \frac{\partial^2 u}{\partial z^2}\frac{\partial z}{\partial x}\right)$$

$$= \frac{\partial^2 u}{\partial v^2} + 4\frac{\partial^2 u}{\partial z \partial v} + 4\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial y} = \frac{\partial u}{\partial v} - \frac{\partial u}{\partial z}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial v^2}\frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial v \partial z}\frac{\partial z}{\partial y} - \left(\frac{\partial^2 u}{\partial z \partial v}\frac{\partial z}{\partial y} + \frac{\partial^2 u}{\partial z^2}\frac{\partial z}{\partial y}\right)$$

$$= \frac{\partial^2 u}{\partial v^2} - 2\frac{\partial^2 u}{\partial v \partial z} + \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial v^2}\frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial v \partial z}\frac{\partial z}{\partial y} + 2\left(\frac{\partial^2 u}{\partial z \partial v}\frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial z^2}\frac{\partial z}{\partial y}\right)$$

$$= \frac{\partial^2 u}{\partial v^2} + \frac{\partial^2 u}{\partial v \partial z} - 2\frac{\partial^2 u}{\partial z^2}$$

با جایگذاری در معادله داریم:

$$\cancel{\frac{\partial^2 u}{\partial v^2}} + 4\frac{\partial^2 u}{\partial z \partial v} + 4\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial v^2} + \frac{\partial^2 u}{\partial v \partial z} - 2\frac{\partial^2 u}{\partial z^2} =$$

$$\cancel{2\frac{\partial^2 u}{\partial v^2}} - 4\frac{\partial^2 u}{\partial v \partial z} + 2\frac{\partial^2 u}{\partial z^2} \quad \Rightarrow 9\frac{\partial^2 u}{\partial v \partial z} = 0 \Rightarrow \frac{\partial u}{\partial z} = h(z)$$

$$\Rightarrow u = \int h(z)\,dz + \Phi(v) \Rightarrow$$

$$u = \varphi(z) + \Phi(v) = \varphi(2x-y) + \Phi(x+y)$$

b) $u_{xx} - 2u_{xy} + u_{yy} = 0 \qquad v = y \qquad z = x + y$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial x} = \frac{\partial u}{\partial z}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial z \partial v}\frac{\partial v}{\partial x} + \frac{\partial^2 u}{\partial z^2}\frac{\partial z}{\partial x} = \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial z \partial v}\frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial z^2}\frac{\partial z}{\partial y} = \frac{\partial^2 u}{\partial z \partial v} + \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial y} = \frac{\partial u}{\partial v} + \frac{\partial u}{\partial z}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial v^2}\frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial v \partial z}\frac{\partial z}{\partial y} + \frac{\partial^2 u}{\partial z \partial v}\frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial z^2}\frac{\partial z}{\partial y}$$

$$= \frac{\partial^2 u}{\partial v^2} + 2\frac{\partial^2 u}{\partial v \partial z} + \frac{\partial^2 u}{\partial z^2}$$

باجایگذاری در معادله :

$$\cancel{\frac{\partial^2 u}{\partial z^2}} - \cancel{2\frac{\partial^2 u}{\partial z \partial v}} - \cancel{2\frac{\partial^2 u}{\partial z^2}} + \frac{\partial^2 u}{\partial v^2} + \cancel{2\frac{\partial^2 u}{\partial v \partial z}} + \cancel{\frac{\partial^2 u}{\partial z^2}} = 0$$

$$\Rightarrow \frac{\partial^2 u}{\partial v^2} = 0 \Rightarrow \frac{\partial u}{\partial v} = h(z) \rightarrow u = \int h(z)\, dv + \varphi(z)$$

$$u = v\, h(z) + \varphi(z) = y\, h(x+y) + \varphi(x+y)$$

C) $y\, u_{xy} = x\, u_{xx} + u_x \qquad v = y \qquad z = xy$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial x} = y\frac{\partial u}{\partial z}$$

$$\frac{\partial^2 u}{\partial x^2} = y\left(\frac{\partial^2 u}{\partial z\partial v}\frac{\partial v}{\partial x} + \frac{\partial^2 u}{\partial z^2}\frac{\partial z}{\partial x}\right) = y^2\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial u}{\partial x\partial y} = \frac{\partial u}{\partial z} + y\left(\frac{\partial^2 u}{\partial z\partial v}\frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial z^2}\frac{\partial z}{\partial y}\right) = \frac{\partial u}{\partial z} + y\frac{\partial^2 u}{\partial z\partial v} + xy\frac{\partial^2 u}{\partial z^2}$$

با جایگذاری در معادله داریم:

$$\cancel{y\frac{\partial u}{\partial z}} + y^2\frac{\partial^2 u}{\partial z\partial v} + \cancel{xy^2\frac{\partial^2 u}{\partial z^2}} = \cancel{xy^2\frac{\partial^2 u}{\partial z^2}} + \cancel{y\frac{\partial u}{\partial z}}$$

$$\Rightarrow y^2\frac{\partial^2 u}{\partial z\partial v} = 0 \Rightarrow \frac{\partial^2 u}{\partial z\partial v} = 0 \qquad \frac{\partial u}{\partial z} = h(z)$$

$$u = \int h(z)\,dz + \Phi(v) = \varphi(z) + \Phi(v) = \varphi(xy) + \Phi(y)$$

تمرینات ۷.۲. صفحه ۹۹۵

۱- درجه حرارت در طول یک میله به طول ۱۰cm را بیابید درصورتیکه درجه حرارت دو سر میله صفر بوده و درجه حرارت اولیه میله برابر یکی از توابع زیر باشد ($c^2 = 1.752$)

a) $u(x,0) = \sin 0.1\pi x$

$$\lambda_n^2 = (1.752)\,\pi^2 n^2/100 \qquad u(x,0) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x = \sum_{n=1}^{\infty} b_n \sin 0.1 n\pi x = \sin 0.1\pi x$$

$$\Rightarrow b_1 = 1 \quad , \quad b_n = 0 \quad n > 1$$

$$\Rightarrow u(x,t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x \, e^{-\lambda_n^2 t} = \sin 0.1\pi x \, e^{-\frac{1.752\pi^2}{100} t}$$

b) $$f(x) = \begin{cases} x & ; \quad 0 < x < 5 \\ 10 - x & ; \quad 5 < x < 10 \end{cases} \qquad \lambda_n^2 = \frac{1.752\, n^2 \pi^2}{100}$$

$$b_n = \frac{2}{10} \int_0^5 x \sin \frac{n\pi}{10} x \, dx + \frac{2}{10} \int_5^{10} (10 - x) \sin \frac{n\pi}{10} x \, dx$$

$$= \frac{2}{10} \left[-x \left(\frac{10}{n\pi}\right) \cos \frac{n\pi}{10} x + \left(\frac{10}{n\pi}\right)^2 \sin \frac{n\pi}{10} x \right]_0^5 +$$

$$\frac{2}{10} \left[-(10 - x) \cos \frac{n\pi}{10} x \left(\frac{10}{n\pi}\right) - \left(\frac{10}{n\pi}\right)^2 \sin \frac{n\pi}{10} x \right]_5^{10} = \frac{40}{\pi^2} \frac{\sin \frac{n\pi}{2}}{n^2}$$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{40}{\pi^2} \frac{\sin \frac{n\pi}{2}}{n^2} \sin \frac{n\pi}{10} x \, e^{-0.01752 (n\pi)^2 t}$$

c) $$f(x) = \begin{cases} x & ; \quad 0 < x < 25 \\ 50 - x & \quad 25 < x < 50 \end{cases}$$

$$b_n = \frac{2}{50} \int_0^{25} x \sin \frac{n\pi}{50} x \, dx + \frac{2}{50} \int_{25}^{50} (50 - x) \sin \frac{n\pi}{50} x \, dx$$

* ادامه دقیقاً مشابه مورد قبلی سؤال است (به عهده دانشجو)

٢ـ هریک از مسائل زیر را حل کنید.

a)

$$u_t - 4u_{xx} = xt \quad ; \quad 0<x<1 \quad , \quad t>0$$

$$u(x,0) = \sin \pi x \qquad 0 \leq x \leq 1$$

$$u(0,t) = t \quad ; \quad u(1,t) = t^2 \qquad t \geq 0$$

$$u = v + w \qquad w = ax + b \longrightarrow \begin{cases} u(0,t) = t = b \Rightarrow b = t \\ u(1,t) = a + b = t^2 \Rightarrow a = t^2 - t \end{cases}$$

$$v(0,t) = 0$$

$$v(1,t) = 0$$

$$v(x,0) = \sin \pi x \qquad w = (t^2 - t)x + t$$

$$u = v + w \Rightarrow v_t - 4v_{xx} = x - xt - 1 \quad (*)$$

$$v(x,t) = \sum_{n=1}^{\infty} G(t) \sin \pi x \;\Big|\; * \Longrightarrow \sum_{n=1}^{\infty} \underbrace{\left[\dot{G}(t) + 4n^2\pi^2 G(t)\right]}_{\gamma(t)} \sin \pi x = x - xt - 1$$

$$\gamma(t) = 2\int_0^1 (x - xt - 1) \sin n\pi x \, dx = 2\left[(1-t)\frac{(-1)^n}{n\pi} + \frac{(-1)^n}{n\pi} - \frac{1}{n\pi}\right]$$

$$= 2\left[\frac{2(-1)^n}{n\pi} + \frac{t(-1)^{n+1}}{n\pi} - \frac{1}{n\pi}\right] = \frac{4(-1)^n}{n\pi} + \frac{2(-1)^{n+1}}{n\pi} t - \frac{2}{n\pi}$$

$$\gamma(t) = \dot{G}(t) + 4n^2\pi^2 G(t) \qquad G_h = c_1 e^{-4n^2\pi^2 t}$$

$$D + 4n^2\pi^2 = 0 \Longrightarrow D = -4n^2\pi^2 \qquad G_p = c_2 t + c_3$$

$$\longrightarrow c_2 + 4n^2\pi^2 (c_2 t + c_3) = \frac{4(-1)^n}{n\pi} + \frac{(-1)^{n+1}}{n\pi} 2t - \frac{2}{n\pi}$$

$$\Longrightarrow 4n^2\pi^2 c_2 = \frac{2(-1)^{n+1}}{n\pi} \Longrightarrow c_2 = \frac{2(-1)^{n+1}}{4n^3\pi^3}$$

$$c_2 + 4n^2\pi^2 c_3 = \frac{4(-1)^n}{n\pi} - \frac{2}{n\pi} \Longrightarrow c_3 = \frac{1}{4n^2\pi^2}\left[\frac{4(-1)^n}{n\pi} - \frac{2}{n\pi} + \frac{2(-1)^n}{4n^3\pi^3}\right]$$

$$G(t) = c_1 e^{-4n^2\pi^2 t} + c_2 t + c_3 \Longrightarrow v(x,t) = \sum_{n=1}^{\infty} \left[c_1 e^{-4n^2\pi^2 t} + c_2 t + c_3\right] \sin n\pi x$$

تمرینات ۹.۲ صفحه‌ی ۱۰۲

۱. مسئله گرما در میله نامتناهی را حل کنید در صورتی که دمای اولیه در میله برابر یکی از توابع زیر باشد.

$$a)\quad f(x) = \begin{cases} x & ; \ 0<x<2 \\ 0 & ; \ x>2 \end{cases} \quad , \ f(-x) = f(x)$$

$$A(w) = \frac{2}{\pi}\int_0^{\infty} f(x)\cos wx\, dx = \frac{2}{\pi}\int_0^{2} x\cos wx\, dx$$

$$= \frac{2}{\pi}\left[\frac{x}{w}\sin wx + \frac{\cos wx}{w^2}\right]_0^2 = \frac{2}{\pi}\left[\frac{2}{w}\sin 2w + \frac{\cos 2w}{w^2} - \frac{\cos 0}{w^2}\right]$$

$$\Rightarrow A(w) = \frac{2}{\pi w}\left[2\sin 2w + \frac{\cos 2w - 1}{w}\right]$$

$$u(x,t) = \int_0^{\infty} \frac{2}{\pi w}\left[2\sin 2w + \frac{\cos 2w - 1}{w}\right]\cos wx\, dw$$

ط)

$$B(w) = \frac{2}{\pi}\int_0^{\infty} f(x)\sin wx\, dx = \frac{2}{\pi}\int_0^{1} 2x\sin wx\, dx + \frac{2}{\pi}\int_1^{2} 2x^2\sin wx\, dx$$

$$= \frac{2}{\pi}\left[-\frac{2x}{w}\cos wx + \frac{2}{w^2}\sin wx\right]_0^1 + \frac{2}{\pi}\left[-\frac{2x^2}{w}\cos wx + \frac{4x}{w^2}\sin wx + \frac{4}{w^3}\cos wx\right]$$

$$= \frac{2}{\pi}\left[\frac{-1}{w}\cos w + \frac{2}{w^2}\sin w\right] + \frac{2}{\pi}\left[-\frac{8}{w}\cos 2w + \frac{8}{w^2}\sin 2w - \frac{4}{w^3}\cos 2w + \frac{2}{w}\cos w - \frac{4}{w^2}\sin w + \frac{4}{w^3}\cos w\right]$$

$$= \frac{2}{\pi}\left[\frac{-2}{w^2}\sin w - \frac{8}{w}\cos 2w + \frac{8}{w^2}\sin 2w - \frac{4}{w^3}\cos 2w + \frac{4}{w^3}\cos w\right]$$

$$u(x,t) = \int_0^{\infty} B(w)\sin wx\, dx$$